# On the extinction of Continuous State Branching Processes with catastrophes

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*Joint work with V. Bansaye and J.C. Pardo Millan.*

We model dynamics of pathogen populations subject to random divisions. Divisions can be the result of the splitting of the pathogen population, of brutal environmental changes, of a medical treatment,... More specifically, this work is motivated by models of cell division with infection (see e.g. [1] for parasite infection and [2] for plasmids infection). In these models, each cell lives for a random exponential time and then splits into two daughter cells. Inside a cell, the pathogen dynamic follows a branching process. When a cell divides, its pathogens are randomly shared between the two daughter cells. Experiments conducted in Tamara's laboratory have shown the existence of an asymmetry between the progeny cells in Escherichia coli (see [3]). This led us to study the effects of the asymmetry of the pathogen sharing between daughter cells on the infection propagation.

Thus, we consider in [4] branching processes in continuous time with continuous state space which undergo random divisions at constant rate. These processes do not satisfy the usual branching property anymore. In this talk, we will develop new methods to study their dynamics and derive their long time behaviour. These processes can either survive with positive probability, or go extinct almost surely. When they survive, we characterize their growth. When they go extinct almost surely, we highlight that three different regimes appear for the speed of extinction. These regimes are determined by thresholds dependent on biological parameters.

We apply our results to infection model introduced in [1] and obtain the speed of the infection propagation in the cell population. This speed depends on the cell division rate, the growth rate of the pathogens, and the way the pathogens are shared between the two daughter cells (via the entropy of this sharing distribution). As expected, we find that the more asymmetric this distribution between the daughter cells is, the fewer infected cells there are. We quantify this relation and identify thresholds in this speed of propagation.

[1] V. Bansaye, V.C. Tran. Branching Feller diffusion for cell division with parasite infection. ALEA Lat. Am. J. Probab. Math. Stat., 8:95–127, 2011.
[2] Kimmel, M. Quasistationarity in a branching model of division-within-division. Classical and Modern Branching Processes (K. B. Athreya and P. Jagers, eds.), 157–164. Springer, New York. 1997.
[3] E.J. Stewart, R. Madden, G. Paul and F. Taddei. Aging and death in a organism that reproduces by morphologically symmetric division. PLoS Biol 3 (2), 2005.
[4] V. Bansaye, J.C. Pardo Millan, C. Smadi. On the extinction of Continuous State Branching Processes with catastrophes. ArXiv:1301.6093